Theorem: A classic unsolved problem in number theory asks if there are infinitely many pairs of "twin primes", pairs of primes separated by 2, such as 3 and 5, 11 and 13, or 71 and 73. There is only one group of triplet primes : 3, 5, 7

Proof: We will prove the theorem by contradiction. Assume we have found a triplet of primes p_1, p_2, p_3 other than $3, 5, 7$. By the definition of a triplet of primes, $p_2 = p_1 + 2$ and $p_3 = p_1 + 4$. Then we have the primes $p_1, p_1 + 2, p_1 + 4$. However, one of these numbers must be divisible by 3. Recall that:

Lemma: For any integer n, one of n, n+2 or n+4 must be divisible by 3.

Proof: Choose an arbitrary integer n. 3 either divides or does not divide n. If $3 \mid n$ *then the chosen n is divisible by 3. If* $3 \nmid n$ *then by the Division Theorem (*)* $n = 3k + r$ *for some k not equal to zero where* $0 < r < 3$*. So r equals 1 or 2.*

Add 2 to both sides of (*): $n + 2 = 3k + (r + 2)$

If $r = 1$ *in (*) then we have* $n + 2 = 3(k + 1)$ *after factoring out 3 which by the definition of divisibility means that* $3 \mid n + 2$*. If* $r = 2$ *then we have* $n + 2 = 3(k + 1) + 1$*: (**)*

Add 2 to both sides of (**): $n + 4 = 3(k + 1) + 3$

Clearly $n+4 = 3(k+2)$ *after factoring so by the definition of divisibility* $3 \mid n+4$*. This covers all cases for an arbitrarily chosen n so the result holds for all n, proving the Lemma.*

Since the primes are a subset of the integers we know one of $p_1, p_1 + 2, p_1 + 4$ is divisible by 3. But they are prime, and none are equal to 3. This is a contradiction which means our assumption is false. So the negation is true:
There is only one group of triplet primes: 3, 5, 7. The theorem is proved.